

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

## **Information for Candidates**

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. This paper has nine questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. The curve C has equation  $y = 2e^x + 3x^2 + 2$ . The point A with coordinates (0, 4) lies on C. Find the equation of the tangent to C at A. (5)

2. Express 
$$\frac{x}{(x+1)(x+3)} + \frac{x+12}{x^2-9}$$
 as a single fraction in its simplest form. (6)

**3.** The functions f and g are defined by

f: 
$$x \alpha \ x^2 - 2x + 3, x \in \mathbb{R}, \ 0 \le x \le 4,$$
  
g:  $x \alpha \ \lambda x^2 + 1$ , where  $\lambda$  is a constant,  $x \in \mathbb{R}$ .  
(a) Find the range of f.  
(b) Given that gf(2) = 16, find the value of  $\lambda$ .  
(3)

4. (a) Sketch, on the same set of axes, the graphs of

$$y = 2 - e^{-x}$$
 and  $y = \sqrt{x}$ . (3)

[It is not necessary to find the coordinates of any points of intersection with the axes.]

Given that  $f(x) = e^{-x} + \sqrt{x} - 2$ ,  $x \ge 0$ ,

(b) explain how your graphs show that the equation f(x) = 0 has only one solution,

(1)

(c) show that the solution of f(x) = 0 lies between x = 3 and x = 4. (2)

The iterative formula  $x_{n+1} = (2 - e^{-x_n})^2$  is used to solve the equation f(x) = 0.

(*d*) Taking  $x_0 = 4$ , write down the values of  $x_1, x_2, x_3$  and  $x_4$ , and hence find an approximation to the solution of f(x) = 0, giving your answer to 3 decimal places. (4)



Figure 1 shows a sketch of the curve with equation  $y = e^{-x} - 1$ .

(a) Copy Fig. 1 and on the same axes sketch the graph of  $y = \frac{1}{2} |x - 1|$ . Show the coordinates of the points where the graph meets the axes. (2)

The x-coordinate of the point of intersection of the graph is  $\alpha$ .

- (b) Show that  $x = \alpha$  is a root of the equation  $x + 2e^{-x} 3 = 0$ . (3)
- (c) Show that  $-1 < \alpha < 0$ . (2)

The iterative formula  $x_{n+1} = -\ln[\frac{1}{2}(3-x_n)]$  is used to solve the equation  $x + 2e^{-x} - 3 = 0$ .

- (d) Starting with  $x_0 = -1$ , find the values of  $x_1$  and  $x_2$ . (2)
- (e) Show that, to 2 decimal places,  $\alpha = -0.58$ . (2)

6.

 $f(x) = x^2 - 2x - 3, x \in \mathbb{R}, x \ge 1.$ 

(a) Find the range of f.	(1)
(b) Write down the domain and range of $f^{-1}$ .	(2)
<ul> <li>(c) Sketch the graph of f<sup>-1</sup>, indicating clearly the coordinates of any point at w intersects the coordinate axes.</li> <li>Given that g(x) =  x - 4 , x ∈ ℝ</li> </ul>	hich the graph (4)
(d) find an expression for $af(x)$	(2)
(a) find an expression for $g_1(x)$ . (c) Solve $g_1(x) = 8$	(2)

7.

$$f(x) = x + \frac{e^x}{5}, \qquad x \in \mathbb{R}.$$

(a) Find f'(x).

The curve *C*, with equation y = f(x), crosses the *y*-axis at the point *A*.

- (b) Find an equation for the tangent to C at A.
- (c) Complete the table, giving the values of  $\sqrt{\left(x + \frac{e^x}{5}\right)}$  to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x+\frac{e^x}{5}\right)}$	0.45	0.91			

- 8. (a) Express  $2 \cos \theta + 5 \sin \theta$  in the form  $R \cos (\theta \alpha)$ , where R > 0 and  $0 < \alpha < \frac{\pi}{2}$ . Give the values of R and  $\alpha$  to 3 significant figures. (3)
  - (b)Find the maximum and minimum values of  $2 \cos \theta + 5 \sin \theta$  and the smallest possible value of  $\theta$  for which the maximum occurs. (2)

The temperature  $T \circ C$ , of an unheated building is modelled using the equation

$$T = 15 + 2\cos\left(\frac{\pi t}{12}\right) + 5\sin\left(\frac{\pi t}{12}\right), \ 0 \le t < 24,$$

where *t* hours is the number of hours after 1200.

(c) Calculate the maximum temperature predicted by this model and the value of t when this maximum occurs. (4)

(d) Calculate, to the nearest half hour, the times when the temperature is predicted to be  $12 \,^{\circ}\text{C}$ . (6)

(2)

(3)

(2)